Correspondence Free Registration through a
Point-to-Model Distance Minimization

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Abstract

This paper presents a novel formulation, which derives in a smooth minimization problem, to tackle the rigid registration between a given point set and a model set. Unlike most of the existing works, which are based on minimizing a point-wise correspondence term, we propose to describe the model set by means of an implicit representation. It allows a new definition of the registration error, which works beyond the point level representation. Moreover, it could be used in a gradient-based optimization framework. The proposed approach consists of two stages. Firstly, a novel formulation is proposed that relates the registration parameters with the distance between the model and data set. Secondly, the registration parameters are obtained by means of the Levengberg-Marquardt algorithm. Experimental results and comparisons with state of the art show the validity of the proposed framework.

1. Introduction

Registration problem has been largely studied in the computer vision community since the last two decades (e.g., [5], [8], [16], [22], [7]). It aims at finding the best transformation that places both the given data set and its corresponding model set into the same reference system as close as possible. The different approaches proposed in the literature can be broadly classified into two categories, depending on whether an initial information is required (*fine registration*) or not (*coarse registration*); see [18] for a detailed survey. The proposed approach lies within the fine rigid registration category.

Typically, the fine registration process consists of iterating the following two stages. Firstly, the correspondence between every point from the current data set and the model set should be found. These correspondences are used to define the residual error of the registration. Secondly, the best

set of parameters that minimizes the sum of these residuals should be found. These two stages are iteratively applied till convergence is reached. The Iterative Closest Point (ICP)—originally introduced by [5] and [8]—is one of the most widely used registration techniques using this two-stage scheme. Since then, several variations and improvements have been proposed in order to increase the efficiency and robustness of the method (e.g., [12], [19], [26], [13]). It should be noticed that the correspondence search in these approaches makes the whole scheme as a discrete evolution.

In order to avoid the correspondence search in the first stage, different techniques have been proposed in the literature: i) Implicit polynomials have been used in [20] to represent both the data set and model set. Then an accurate pose estimation is computed through constructing two co-variants. ii) Probabilistic representations have been used to describe both data set and model set (e.g. [23], [14], [7]). iii) In [10] the point-wise problem is avoided by using a distance field of the model set; the value and behavior of this distance field is computed in a discrete domain. iv) In [15] the behavior of the distance field is approximated analytically based on the curvature information. v) An implicit polynomial is used in [24] to fit the distance field, which later defines a gradient field leading the data set towards that model set.

The current paper proposes a novel formulation different to the point-wise based correspondence approaches. It is based on an implicit representation of the model set that allows to define a continuous and smooth distance function for the registration. Hence, it is independent of point densities and robust to noise and missing data. The main contribution of the proposed approach is the novel way of distance definition, which avoids correspondence search of the classical registration algorithms. Moreover it is differentiable with respect to the registration parameters and allows solving the registration problem through a gradient based optimization algorithm.
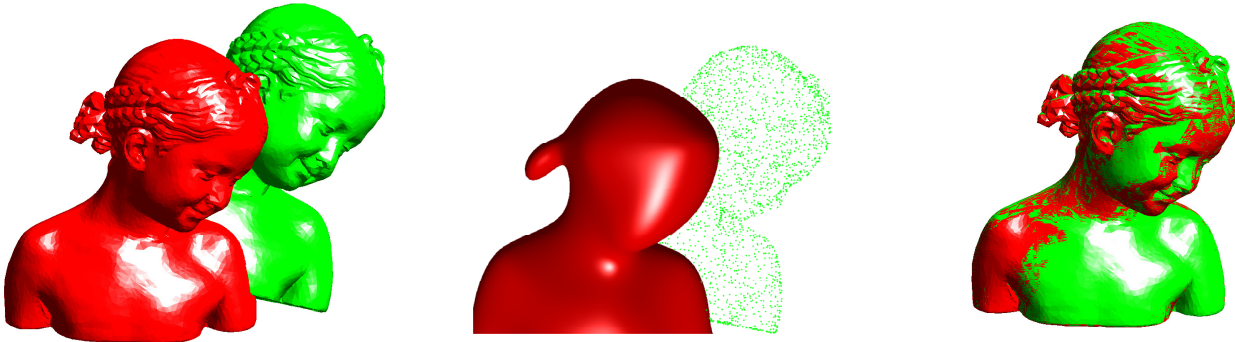
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Figure 1. $(left)$ Initial positions of data and model sets. $(middle)$ Data points and 7th. degree IP describing the model set. $(right)$ Result from the proposed approach. In $(left)$ and $(right)$ sets of points are represented through triangular meshes to highlight details.

The remainder of this paper is organized as follows. Section 2 introduces related works together with their advantages and disadvantages. Section 3 presents the proposed registration approach based on a non-linear minimization of the distance between the given data set and an implicit representation of the model. Experimental results and comparisons with state of the art are presented in Section 4. Finally, conclusions and future work are given in Section 5.

2. Motivations and Related Work

This section describes major and recent contributions in the correspondence free registration category. Most of these methods use different representations to model the registration error. Probabilistic approaches represent each given set by a probabilistic model like multivariate t-distributions [23] or mixture of Gaussians [14, 9]; hence, the registration problem becomes a problem of aligning the two mixtures. In [14] a closed-form expression for the L_2 distance between two Gaussian mixtures is proposed. Similarly, in [7] a smooth distance function based on the mixture of Gaussians is proposed. The modelled distance function is used in the quasi-Newton algorithm to find the optimal rigid parameters.

Probabilistic approaches based on mixture models are highly dependent on the number of mixtures used for modelling the sets. This problem is generally solved by assuming a user defined number of mixtures or as many as the number of points. The former scheme needs the points to be clustered, while the latter one results in a very expensive optimization problem that cannot handle large data sets or could get trapped in local minimum when complex sets are considered. Generally speaking, although these methods do not require any correspondence search, all points in the model set are implicitly considered as a potential correspondence for each single point in the given data set.

On the contrary to the previous approaches, [24] proposes a fast registration method based on solving an energy minimization problem derived from an implicit polynomial (IP) fitted to the given model set [25]. This IP is used to define a gradient flow that drives the data set to the model set without using point-wise correspondences. The energy functional is minimized by means of a heuristic two step process. Firstly, every point in the given data set moves freely along the gradient vectors derived from the IP. Secondly, the outcome of the first step is used to define a single transformation that represents this movement in a rigid way. These two steps are repeated alternately until convergence is reached. The weak point of this approach is the first step that lets the points move independently in the proposed gradient flow. Furthermore, the proposed gradient flow is not precise, specially close to the boundaries.

The approach presented in [10] overcomes the non-differentiable nature of ICP by using a derivable distance transform—Chamfer distance. The error function derived from that distance field is a smooth function, and its derivatives can be analytically computed; hence it can be minimized through the Levenberg-Marquardt algorithm (LMA) to find the optimal registration parameters. The main disadvantage of [10] is the precision dependency on the grid resolution, where the Chamfer distance transform and discrete derivatives are evaluated. Hence, this technique cannot be directly applied when the point set is sparse or unorganized.

The distance field used in [10] is a discrete field and its derivatives used in LMA are not precise enough. In [15] the authors present a local quadratic approximation of the distance function based on the curvature information. These local approximations define the distance field of the model points, and reformulate the registration problem as an optimization problem which can be solved by Newton's method. Unfortunately, this method needs curvature information of the point set to find each local approximation, hence it is computationally expensive and sensitive to noise.

In this work we use implicit representations to reformulate the point-to-point registration as a point-to-model problem. Firstly, the model set is described with an IP, and then

the approximated distance between the data set and the fitted polynomial is minimized to find the best rigid transformation. Figure 1(*middle*) shows a seventh degree IP that is considered instead of the model set. It should be mentioned that the IP is just used as an interface to tackle the registration problem. For instance, in the extreme case, when the data set $\mathbf{P}$ is a rigid transformation of the model set $\mathbf{Q}$ and $f_\mathbf{c}$ is the best polynomial fitting the model set, then it could be proved that:

$$min_{\mathbf{R},\mathbf{t}} Dist(\mathbf{R}\mathbf{P} + \mathbf{t}, f_\mathbf{c}) = Dist(\mathbf{Q}, f_\mathbf{c}),$$

where $Dist$ refers to the orthogonal distance of a set of points to the implicit polynomial; and $[\mathbf{R}, \mathbf{t}]$ refers to the rotation and translation of the rigid transform.

3. Proposed Approach

The proposed approach consists of two main steps. The first step formulates the registration error based on the approximated distance between the current data set and the implicit function used for representing the model set. This formulation relates the error function with the registration parameters. The second step finds the optimal rigid parameters that minimize the proposed registration error through its gradient information. Before detailing the aforementioned two steps IP fitting is introduced, which is used to describe the model set. It should be noticed that the proposed formulation is valid for any implicit representation (e.g. implicit RBF and B-Splines).

3.1. Model Representation

This section briefly describes the algorithm used for finding the implicit polynomial that fits the model set. The 3L algorithm [6] has been chosen due to its simplicity and robustness; other algebraic or geometric fitting approaches could be adopted since the proposed distance formulation is independent of the algorithm used for fitting the model. Finding the *best* IP representation is out of the scope of the current paper and several approaches can be found in the literature (e.g., [3], [4], [6], [17], [21], [25]).

Without loss of generality, lets consider the 2D case, where an implicit polynomial describes a set of points in the plane fulfilling this equation:

$$f_\mathbf{c}(x, y) = \sum_{\substack{(i+j) \leqq d \\ \{i,j\} \geqq 0}} c_{i,j} x^i y^j = 0, \qquad (1)$$

which could be represented in the vector form:

$$f_\mathbf{c}(\mathbf{x}) = \mathbf{m}^{\mathrm{T}} \mathbf{c} = 0, \qquad (2)$$

where $\mathbf{m}$ is the column vector of monomials and $\mathbf{c}$ is the polynomial coefficient vector; the fitting problem consists

in first defining a criterion—or residual error—to measure the distance between the zero set and the given data set, and then minimizing this criterion to find the best coefficient vector $\mathbf{c}$.

The simplest and straightforward criterion could be the sum of squared IP values at the given data points (i.e. $\sum_i f_\mathbf{c}(x_i, y_i)^2$). The parameters can be easily obtained by minimizing this criterion, but there is not a clear geometric interpretation and the least square solution for this problem could be very unstable.

To address the two problems mentioned above, the authors in [6] have proposed the 3L algorithm, which consists in generating two additional *level sets*: $\Gamma_{-\delta}$ and $\Gamma_{+\delta}$ from the original data set Γ_0. These two additional data sets are generated so that one is internal and the other is external. These sets are placed at a distance $\pm\delta$ from the original data along a direction that is locally perpendicular to the given data set. In the current implementation a principal components analysis (PCA) based approach, in a local neighborhood for every point, has been used for estimating this direction. Hence, the 3L algorithm incorporates a control for a local continuity resulting in a more stable solution.

The 3L fitting algorithm is then formalized as a linear least squares *explicit* polynomial fitting problem. Considering the three level sets: $\{\Gamma_{-\delta}, \Gamma_0, \Gamma_{+\delta}\}$ the equation (2) is now defined by using a block matrix $\mathbf{M}_{3L}$ and a block column vector $\mathbf{b}$:

$$\mathbf{M}_{3L} = \begin{bmatrix} \mathbf{M}_{\Gamma_{-\delta}} \\ \mathbf{M}_{\Gamma_0} \\ \mathbf{M}_{\Gamma_{+\delta}} \end{bmatrix}, \mathbf{b} = \begin{bmatrix} -\epsilon \\ \mathbf{0} \\ +\epsilon \end{bmatrix}, \qquad (3)$$

where $\mathbf{M}_{\Gamma_0}, \mathbf{M}_{\Gamma_{+\delta}}, \mathbf{M}_{\Gamma_{-\delta}}$ are matrices of monomials calculated in the original, inner and outer set respectively; $\pm\epsilon$ are the corresponding expected values in the inner and outer level sets. Then, the least squares solution for $\mathbf{c}$ is obtained:

$$\mathbf{c} = \mathbf{M}_{3L}^{\dagger} \mathbf{b} = (\mathbf{M}_{3L}^T \mathbf{M}_{3L})^{-1} \mathbf{M}_{3L}^T \mathbf{b}, \qquad (4)$$

where $\mathbf{M}_{3L}^{\dagger}$ denotes the pseudoinverse of $\mathbf{M}_{3L}$.

3.2. Distance Formulation

The registration process seeks for the best transformation parameter $\mathbf{\Theta}$ which contains rotation $\mathbf{R} = \mathbf{R}_\theta$ and translation $\mathbf{t} = [\mathbf{t_x}, \mathbf{t_y}]^{\mathrm{T}}$ in rigid case. The optimal parameter moves the data set $\mathbf{P} = \{p_i\}_1^N$, in a rigid way, as close as possible to the model $f_\mathbf{c}(\mathbf{x})$:

$$\hat{\mathbf{\Theta}} = argmin_{\mathbf{\Theta}} \left(\sum_{i=1}^N Dist^2(\mathbf{R}p_i + \mathbf{t}, f_\mathbf{c}) \right), \qquad (5)$$

for this purpose, the distance function, $Dist$, between the data set and the model should be approximated. In the current work, the estimation of the orthogonal distance proposed in [21] is used. This approximation is based on the

first order Taylor expansion of the distance function. It has some interesting properties including: i) independence of the zero set representation; and ii) invariance to rigid body transformation. It is computed through normalizing the algebraic distance by the gradient norm:

$$Dist(p, f_{\mathbf{c}}) \approx \frac{|f_{\mathbf{c}}(p)|}{||\nabla f_{\mathbf{c}}(p)||}, \qquad (6)$$

using this approximation in (5) the registration parameters can be found by minimizing the following function:

$$Dist_{\boldsymbol{\Theta}} = \sum_{i=1}^{N} \left(\frac{f_{\mathbf{c}}(\mathbf{R}p_i + \mathbf{t})}{\| \nabla f_{\mathbf{c}}(\mathbf{R}p_i + \mathbf{t}) \|} \right)^2 \qquad (7)$$

$$= \sum_{i=1}^{N} (w_i f_{\mathbf{c}}(\mathbf{R}p_i + \mathbf{t}))^2 = \sum_{i=1}^{N} d_i^2,$$

where:

$$d_i = w_i f_{\mathbf{c}}(\mathbf{R}p_i + \mathbf{t}), \qquad (8)$$
$$w_i = 1/\| \nabla f_{\mathbf{c}}(\mathbf{R}p_i + \mathbf{t}) \|,$$

show the distance of each item and the weight to approximate this distance. Thus the point-to-point registration will be done in a higher level using a curve or surface as an interface. It will provide a rich structure as well as many advantages like robustness to noise and missing data.

3.3. Distance Optimization

The distance presented above provides a correspondence free formulation for the registration problem, which is directly related to rigid parameters. This relation could be exploited in many optimization algorithms. Here we use gradient based algorithms like gradient descent and Levenberg−Marquardt algorithm (LMA). The gradient information shows the sensitivity of distance with respect to rigid parameters as illustrated in Fig. 2.

LMA is a well-known technique in non-linear optimization [11], which is particularly proposed for functions in the form of sum of squared residuals as the case in (7). This method proposes a tradeoff between two well known methods in nonlinear optimization: the Gauss−Newton algorithm and the gradient descent algorithm. In order to handle LMA, the value of the function (7) and its partial derivatives, which are expressed in a *Jacobian* matrix J, should be provided. Since LMA uses the gradient information of the objective function, the first order distance approximation in (7) captures this information; hence, better approximations would not benefit the result of LMA. It should be mentioned that the derivatives of (7) must be calculated with respect to the parameters $\boldsymbol{\Theta} = [\theta \quad \mathbf{t_x} \quad \mathbf{t_y}]^{\mathrm{T}}$, where $\theta, \mathbf{t_x}$ and $\mathbf{t_y}$ capture the three degrees of freedom of the rigid registration. Hence, the first column of the *Jacobian* matrix can be computed as follow:
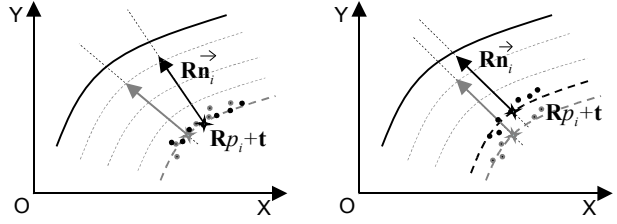

Figure 2. ($left$) Sensitivity of the distance with respect to small changes in rotation. ($right$) Sensitivity of the distance with respect to the translation along y axis.

$$J(i,1) = \frac{\partial d_i}{\partial \theta} = (\partial w_i/\partial \theta) f_{\mathbf{c}}(\mathbf{R}p_i + \mathbf{t}) + w_i \frac{\partial f_{\mathbf{c}}(\mathbf{R}p_i + \mathbf{t})}{\partial \theta}, \qquad (9)$$

since the implicit function $f_{\mathbf{c}}$ is a smooth function, w_i could be considered as a constant weight, then the first term could be ignored:

$$J(i,1) = w_i(\mathbf{R}'_\theta p_i . \nabla f_{\mathbf{c}}(\mathbf{R}p_i + \mathbf{t})), \qquad (10)$$

where $\mathbf{R}'_\theta$ is the derivative of the rotation matrix w.r.t. the rotation angle, and $\nabla f_{\mathbf{c}}$ is the gradient with respect to (x, y) components. Similarly, other columns of the Jacobian matrix can be calculated as:

$$J(i,2) = \frac{\partial d_i}{\partial \mathbf{t_x}} = w_i \frac{\partial}{\partial x} f_{\mathbf{c}}(\mathbf{R}p_i + \mathbf{t}), \qquad (11)$$

$$J(i,3) = \frac{\partial d_i}{\partial \mathbf{t_y}} = w_i \frac{\partial}{\partial y} f_{\mathbf{c}}(\mathbf{R}p_i + \mathbf{t}).$$

For the 3D case the Jacobian matrix includes six columns corresponding to three rotation and three translation parameters. As a general formula each entry of this matrix could be computed as:

$$J(i,j) = w_i \left(\frac{\partial}{\partial \boldsymbol{\Theta_j}}(\mathbf{R}p_i + \mathbf{t}) \right) . \nabla f(\mathbf{R}p_i + \mathbf{t}), \qquad (12)$$

where $\boldsymbol{\Theta_j}$ is the jth parameter of $\boldsymbol{\Theta} = [\theta, \phi, \psi, \mathbf{t_x}, \mathbf{t_y}, \mathbf{t_z}]$.

Having estimated the proposed distance (7) and its Jacobian matrix through (9), (10) and (11) it is easy to perform LMA in order to refine the rigid parameters $\boldsymbol{\Theta}$:

$$\boldsymbol{\Theta}^{k+1} = \boldsymbol{\Theta}^k + \beta \triangle \boldsymbol{\Theta},$$
$$(J^{\mathrm{T}}J + \lambda diag(J^{\mathrm{T}}J)) \triangle \boldsymbol{\Theta} = J^{\mathrm{T}}D, \qquad (13)$$

where β is the refinement step; $diag(J^{\mathrm{T}}J)$ is the diagonal matrix containing the elements of $(J^{\mathrm{T}}J)$; $\triangle \boldsymbol{\Theta}$ represents the refinement vector for the rigid parameters; λ is the damping parameter in LMA; and the vector D is a column vector containing $Dist(\mathbf{R}p_i + \mathbf{t}, f_{\mathbf{c}})$, $\mathbf{R}$ and $\mathbf{t}$ are the current rotation and translation respectively. In the current implementation they are initialized as $\theta = 0$, $\mathbf{t_x} = 0$ and $\mathbf{t_y} = 0$; more evolved initializations, such as using simple SVD based techniques, could be used since we are tackling the rigid registration case. Parameter refinement (13) must be repeated till convergence is reached.
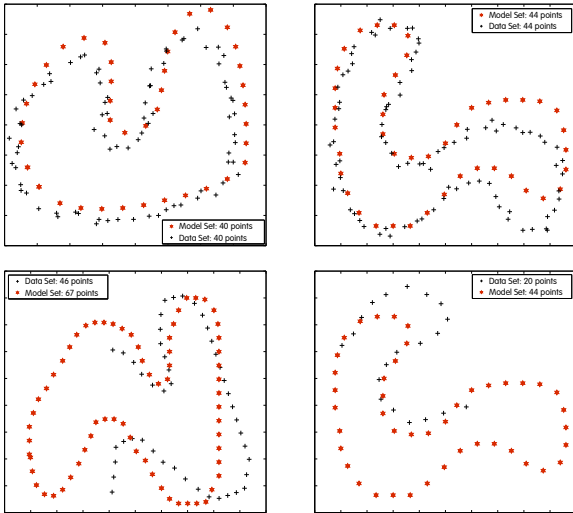
Figure 3. Initial positions of data sets and model sets for noisy (*top*) and partial overlap (*bottom*) examples registered with the different approaches.
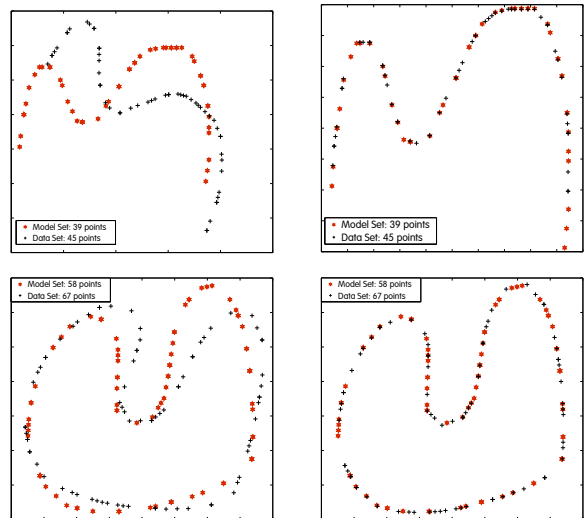

Figure 4. Model sets and data sets containing different density of points. (*left*) Initial configurations. (*right*) Final results from the proposed approach.

4. Experimental Results and Comparisons

The proposed approach has been evaluated using different data sets and model sets. Additionally four techniques (i.e., [14], [10], [24] and [15]) from the state of the art, together with the classical ICP [5], have been implemented for a comparative study in the 2D and 3D cases. Each techniques iterates till one of the stopping criteria is reached: maximum number of iterations (#Iter=30) or relative registration error smaller than a given threshold. The relative registration error is defined as: $\epsilon = |E_t - E_{t-1}|/E_t$, where E_t refers to the error between the model and data set at iteration t. In our implementation ($\epsilon < 0.001$) has been used.

On the contrary to the relative registration error, which is an internal measure, an *Accumulated Residual Error* (ARE) is used during the comparisons. It is computed by measuring the accumulated error, in a point-wise manner, from the data set to a *reference set*. This reference set corresponds to a highly detailed description of the model set. It contains the model set and on average is defined by a set of points ten times larger than the model set. Each residual error is computed by finding the nearest point in between the registered data set and the reference set.

Figure 3 shows initial configurations for four different data and model sets. The first row corresponds to closed contours with a full overlap. Data sets have been obtained by rotating and translating the corresponding model set, and by adding Gaussian noise to study the robustness of all the techniques. Accuracy and number of iterations are provided in Table 1. It should be highlighted that the proposed approach converges in all the cases and most of the time with the smallest error and lowest number of iterations, in spite

of the noise in the data set. In these examples IPs of degree six have been used for fitting the model sets. The IP degree could be automatically determined through the algorithm in [25], which is based on the QR decomposition of the monomial matrix. Figure 3 (*bottom*) presents two examples where data set partially overlaps the corresponding model set; data and model sets correspond to uniform sampling of different boundaries. Model sets have been fitted by sixth degree IPs in both cases. Both of them have been registered using the proposed technique and the five aforementioned ones; the obtained registration accuracy is given in the third and fourth rows of Table 1, as well as the number of iterations when one of the stopping criteria is reached.

Figure 4 presents challenging situations where model sets and data sets contain different densities of points. Fig. 4(*left*) shows the initial configurations while Fig. 4(*right*) depicts the results obtained by using the proposed approach. Quantitative results from these two examples are presented in Table 1. The challenge in these examples lie on the non-existence of any point to point correspondence, although both clouds of points correspond to the same contour. The proposed approach, since the model set is represented by a unified IP, is robust in this kind of situations.

In addition to 2D cases presented above, 3D real objects from public data sets ([1] and [2]) have been registered with the proposed approach and compared with state of the art techniques. The illustration presented in Fig. 1 corresponds to a data set defined by 811 points. The model set contains 926 points and is represented by means of a seventh degree IP. The result obtained with the proposed approach is shown in Fig. 1(*right*). Quantitative information about

Table 1. Comparisons of registration results for 2D cases (ICP: Iterative Closest Point; GMM: Gaussian Mixture Models; DT: Distance Transform; GF: Gradient Flow; DA: Distance Approximation; PA: Proposed Approach).

| Figure	ICP [5]		GMM [14]		DT [10]		GF [24]		DA [15]		PA	
	ARE	#Iter	ARE	#Iter	ARE	#Iter	ARE	#Iter	ARE	#Iter	ARE	#Iter
Fig. 3(top-$left$)	1.62	*13*	2.32	*13*	1.65	*25*	1.64	*27*	1.65	*11*	**1.59**	*4*
Fig. 3(top-$right$)	1.41	*10*	1.34	*7*	1.42	*28*	1.42	*15*	1.40	*9*	**1.32**	*6*
Fig. 3($bot.$-$left$)	0.91	*7*	5.11	*10*	4.00	*30*	0.98	*16*	0.92	*10*	**0.53**	*9*
Fig. 3($bot.$-$right$)	0.52	*13*	1.75	*18*	0.20	*27*	0.29	*20*	0.35	*17*	**0.18**	*15*
Fig. 4(top)	0.26	*14*	0.89	*10*	0.39	*29*	0.48	*12*	0.42	*13*	**0.19**	*11*
Fig. 4($bottom$)	1.54	*16*	2.48	*13*	0.57	*30*	1.92	*28*	1.22	*12*	**0.34**	*13*
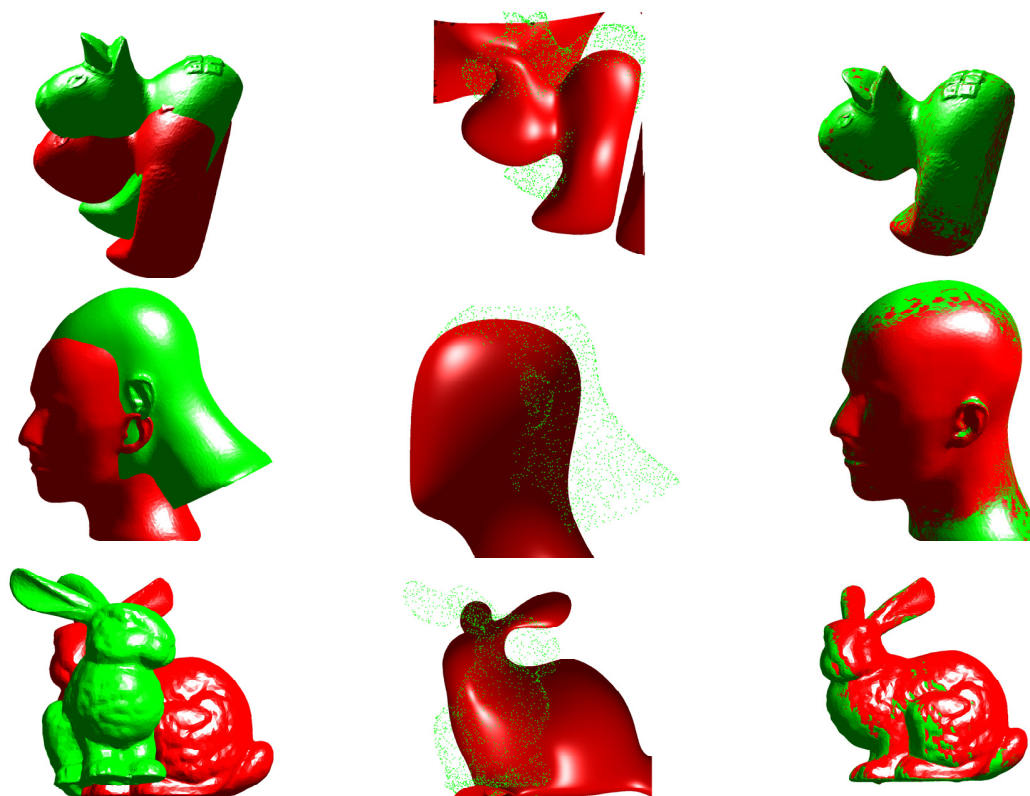
Figure 5. Real data sets (from [1] and [2]) registered with the proposed approach and state of the art techniques. ($left$) Initial set up of the given data and model sets represented by means of triangular meshes to highlight details. ($middle$) IPs representing model sets and data points. ($right$) Results of the proposed registration approach represented through triangular meshes to make easier a visual evaluation.

the registration process, as well as comparisons with other approaches are provided in Table 2; the stopping criteria considered in Table 1 is also used here.

Figure 5 presents three additional experimental results using 3D real data sets; Figure 5($left$) shows initial position of data and model sets both represented by means of triangular meshes to highlight the details. Figure 5($middle$) depicts IPs describing model sets together with the points of their corresponding data sets. A seventh degree IP is used in the (top) row to represent the 745 points of the model

set, while the data set contains 609 points. A fifth degree IP is used in the ($middle$) row, in this case the data set contains 625 points while the model set is defined by 639 points. Finally, a sixth degree IP is used to describe the 817 points of the model of the example presented in the ($bottom$) row; in this case the data set contains 724 points. Figure 5($right$) presents the registration obtained with the proposed approach. Statistics about their registration process and comparisons with state of the art techniques are presented in Table 2.
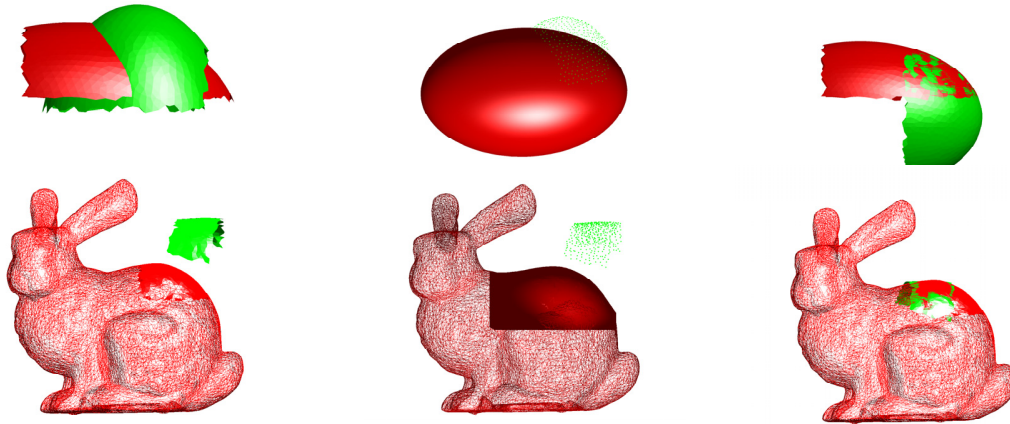
Figure 6. Partial overlap cases. ($left$) Initial set up of data sets and model sets to be registered. ($middle$) IPs representing model sets and data points from the data sets. ($right$) Results from the proposed approach.

Table 2. Comparisons of registration results for 3D cases (ICP: Iterative Closest Point; GMM: Gaussian Mixture Models; DT: Distance Transform; GF: Gradient Flow; DA: Distance Approximation; PA: Proposed Approach).

	ICP [5]		GMM [14]		DT [10]		GF [24]		DA [15]		PA	
Figure	ARE	#Iter	ARE	#Iter	ARE	#Iter	ARE	#Iter	ARE	#Iter	ARE	#Iter
Fig. 1	77.19	20	147.19	30	85.05	14	75.71	26	77.32	13	**75.72**	10
Fig. 5(top)	63.69	16	112.55	30	63.71	6	61.61	11	64.14	14	**61.52**	8
Fig. 5($middle$)	53.45	26	108.21	30	52.70	9	42.39	29	53.90	15	**42.75**	10
Fig. 5($bottom$)	49.25	24	124.58	30	53.15	15	46.49	28	48.14	18	**46.09**	13
Fig. 6(top)	456.64	30	139.77	30	109.29	30	1364.6	30	146.54	30	**7.40**	27
Fig. 6($bottom$)	42.20	30	185.28	30	36.09	30	32.01	30	47.80	30	**29.30**	30

Finally, two cases where model sets and data sets are partially overlapped are presented in Fig. 6. The (top) row shows a simple example where the data set (860 points) and model set (835 points) are picked from the same ellipsoid, which is described by a second degree IP in the presented approach. These two sets are partially overlapped (about 40%) as shown in the last column. Despite the simplicity of the problem, none of the techniques presented in Table 2, except our approach, converge to the right configuration. All these registration techniques are trapped in a local minimum, while our approach exploits the extrapolation provided by the fitted surface. The ($bottom$) row presents another illustration of partial overlap. In this case, although all the techniques have similar behavior, the proposed approach has the smallest ARE.

The evolution of ARE for registering Fig. 5($bottom$) is illustrated in Fig. 7. It can be appreciated that the proposed approach has the smallest ARE and the fastest convergence. Although GF [24] reaches the same optimal ARE its convergence is slower; the oscillation in DT [10] is due to the discrete approximation of the distance field, which is not the
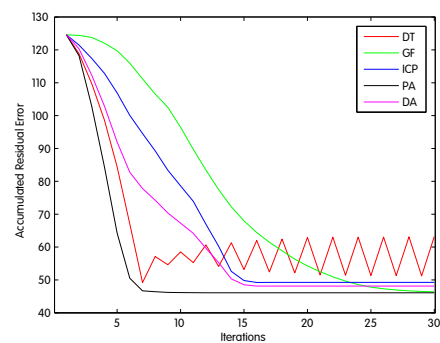

Figure 7. Evolution of ARE of different registration algorithms along 30 iterations.

case of the proposed approach that has a smooth behavior.

5. Conclusions and Future Work

In this paper a novel accurate registration distance is presented, which is based on a correspondence free formula-

tion. Additionally, its continuous nature allows the use of gradient based optimization frameworks. Experimental results and comparisons show both fast convergence and robustness in challenging situations (e.g., noisy data; partial overlap between data set and its corresponding model set; different densities). As a future work the proposed registration error will be extended for more general implicit representations in a non-rigid deformation space. Furthermore, since the proposed formulation is independent of the fitting, a coarse to fine representation would be explored to avoid local minima.

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